Chapter 7 Statics
LECTURE OUTLINE & NOTES

STATICS, p. 119
Forces applied to a body may not cause acceleration; analysis of such situations is statics.

INTRODUCTORY CONCEPTS IN MECHANICS, p. 120
Newton's Laws of Motion

> Newton's second law states that force equals mass times acceleration; this chapter deals with cases in which the acceleration is zero. His third law states that the forces exerted by one body on a second body are equal and opposite to the forces exerted by the second body on the first; this law is not as simple as it looks.

Newton's Law of Gravitation

> The gravitational attraction between two objects is proportional to the product of their masses and the inverse of the square of the distance between them; for objects near the surface of the Earth this force simplifies to the product of the object's mass and *g*, the Earth's gravitation (formulas p. 120).

Dimensions and Units of Measurement

> The fundamental units of mass, length, and time can be variously combined to express every other quantity in mechanics; various systems exist so use care in expressing units (table p. 121).

VECTOR GEOMETRY AND ALGEBRA, p. 121

> A vector has both magnitude and direction; it is visualized as an arrow with length and orientation.

Addition and Subtraction

> Two vectors are added as visualized by placing the tail of arrow one to the head of the other; they are subtracted by adding the subtracted vector multiplied by -1 (below) (figures and formulas pp. 121-122).

Multiplication by a Scalar

> The magnitude of the product of a scalar and a vector is the product of the two magnitudes; the direction of the product is determined by the direction of the vector and the sign of the scalar; a negative sign reverses the vector's direction (formulas p. 122).

Dot Product

> The product of two vectors is the scalar product of their magnitudes and the cosine of the angle between them (formulas p. 123).

Unit Vectors and Projections

> A vector may be expressed as the product of its magnitude and a unit vector; dot-multiplying a vector by a unit vector in the direction of a line makes it possible to evaluate the vector projection onto that line (figure and formulas pp. 123-124).

Vector and Scalar Equations

> Expressing vectors as the product of magnitudes and unit vectors simplifies isolating magnitude relationships from a vector relationship (figure and formulas p. 124).

The Cross Product

The magnitude of the cross product of two vectors equals the product of their two magnitudes and the sine of the angle between them; the direction of the product is perpendicular to the plane of the two original vectors and in the direction of the advancement of a right hand screw being turned from the first toward the second of the two original vectors (figure and formulas p. 125).

Rectangular Cartesian Components

Three mutually perpendicular unit vectors can be used to project a vector onto rectangular Cartesian components; useful formulas result (formulas p. 126).

FORCE SYSTEMS, p. 127

An accounting system is needed to understand the cumulative effect of multiple forces on a body.

Types of Forces

It may or may not be accurate to simplify analysis of a distributed force by considering it as being applied at a single point of application; forces may act on the body's surface or throughout its mass.

Point of Application and Line of Action

A force vector acting on a body defines a line of action through the point of application.

Moments of Forces

The moment of a force about any point is the cross product of the force and the position vector from the point to any point on the line of action of the force; for an axis of rotation, the moment is a measure of the rotation caused by the force, and can be expressed in terms of the moment about a point on the axis and a unit vector along the axis (figures and formulas pp. 128-129).

Resultant Forces and Moments

The sum of several forces, each with its own line of action, is the resultant force; the sum of the cross products of the forces and the position vectors from one point to any points on the forces's lines of action is the resultant moment about that single point (formulas pp. 129-130).

Couples

A pair of forces may have zero resultant force but nonzero resultant moment, for instance if they have separate lines of action; in that case they are called a couple.

Moments about Different Points

The resultant moment of a set of forces about a first point is equal to the sum of their resultant moment around a second point plus the moment their resultant would have about the first point if the line of action of the resultant passed through the second point (figure and formulas p. 130).

Equivalent Force Systems

If two sets of forces have the same resultant force and the same resultant moment about a point, they are termed equivalent; they will also have the same resultant moment about any other point.

EQUILIBRIUM, p. 131

For any one of the external forces acting on a material element of a system of bodies, the moment about any point equals the cross product of the position vector from that point to that element and the product of the mass and acceleration of the element; the sums include all of the external forces and material elements, so in static equilibrium the accelerations, resultant of all external forces, and resultant moment of those forces about any point are all zero (formulas p. 131).

Free-Body Diagrams

In defining an equilibrium relationship, a free-body diagram can be used to show which body part, body, or body system is part of the relationship, and what outside forces are acting on the part or parts (figures p. 132).

Equations of Equilibrium

Free-body diagrams of isolated elements can lead to understanding of an overall system in equilibrium.

TRUSSES, p. 138

A truss is defined as a structure made up of pieces connected at flexible joints; the pieces can transmit forces of tension or negative tension (compression) along their axes.

Equations from Joints

By isolating the portion of a truss around one joint as a free-body, forces in those members can be determined; by taking those values and working progressively through the truss, remaining forces can be evaluated (figures and formulas pp. 138-139).

Equation from Sections

By finding the free-body of a section of a truss with an axis having only one unknown force with nonzero moment, the rest of the forces making up the resultants of force and moment can be found, because both resultants are zero in static equilibrium (figures and formulas pp. 140-142).

COUPLE-SUPPORTING MEMBERS, p. 142

Unlike a truss element, a rigid member may carry loads that do not run along its axis; lateral forces with nonzero moments across a section complicate analysis.

Twisting and Bending Moments

Taking a section through the member, isolating a free-body on each side, and finding the forces and moments reacting at the section makes it possible to evaluate forces and moments of the member (figures and formulas pp.143-145).

SYSTEMS WITH FRICTION, p. 145

For two bodies in contact, resistance to their sliding movement along a tangent to the surface of contact is approximately proportional to the force they exert on each other (figure and formulas pp. 145-146).

DISTRIBUTED FORCES, p. 148

Forces that act throughout a volume, such as gravity, or over a surface, such as water pressure, require computational summation for evaluation (formulas pp. 148-150).

Single Force Equivalents

Instead of evaluating integrals, some distributed forces can be found by using the concepts of the center of mass or centroids of volume, surface, or a line.

Center of Mass and Center of Gravity

Finding the moment about any point of a force acting equally throughout a body makes it possible to determine the intersection of the line of action of the resultant force and a position vector from that point; such an intersection is the center of mass and its resultant moment is zero regardless of the direction from which the line of action passes through it (figure and formulas pp. 151-152).

Centroids

For bodies of uniform density, the centroids of volume, surface area or a line segment may be found using a formula similar to that for the center of mass but substituting appropriate units; a composite centroid may be found from a summation using the known centroids of several volumes, areas, or line segments (formulas pp. 153-155).

Second Moments of Area

Forces on a plane area that vary linearly with distance from a zero-force line on the plane have a resultant force with a resultant moment about a point on the zero-force line that may be expressed in terms of the rectangular Cartesian components; integrals in these expressions are the second moments of area (figures and formulas pp.156-158).

Parallel Axis Formulas

A known value for one of the second moments of area may be used to find another value for a different origin (figure and formulas p. 158).