Chapter 9 Mechanics
LECTURE OUTLINE & NOTES

MECHANICS OF MATERIALS, p. 261

Three primary aspects of problem solving in the mechanics of materials are:
1) analyzing the equilibrium of forces in the static state
2) finding the relationship of the applied forces to the deformation of a structure, and
3) determining the compatibility of those deformations with structural integrity.

AXIALLY LOADED MEMBERS, p. 262

A member under axial force will typically first deform in a linear relationship between stress and strain; beyond a yield point strain will increase greatly and unloading may not restore the original form (figures and formulas pp. 262-263).

Modulus of Elasticity

At stresses below the yield point, the ratio of stress to strain is called the modulus of elasticity; the product of the load and the original length of a member divided by the product of the area under load and the modulus of elasticity is the change in length (formulas p. 263).

Poisson's Ratio

A member under tension will elongate along its axis and contract in the lateral dimensions; the ratio of lateral strain to longitudinal strain is Poisson's ratio (formula p. 266).

Thermal Deformations

Thermal strain equals the product of the linear coefficient of thermal expansion and the change in temperature; total strain is the sum of thermal strain and the strain from applied loads, and total deformation is the sum of thermal deformation and applied force deformations (formulas p. 267).

Variable Load

Where load is a function of the length of the member, deformation varies continuously with change in length (formulas p. 268).

THIN-WALLED CYLINDER, p. 268

Tangential stress in the walls of a cylinder under pressure equals the product of the pressure and the inner diameter divided by twice the wall thickness; axial stress equals the product of the pressure and the inner diameter divided by four times the wall thickness (figures and formulas p. 269).

GENERAL STATE OF STRESS, p. 270

A force acting over an area of a section through a body has a normal stress component perpendicular to the plane of the section and two shear stress components parallel to the plane and perpendicular to each other; for an element in equilibrium the state of stress at any point can be expressed in terms of three mutually perpendicular normal stresses and three mutually perpendicular shear stresses (figures and formulas pp. 270–271).

PLANE STRESS, p. 271

Considering only stresses in a plane reduces one of the normal and two of the shear stresses to zero.

Mohr's Circle—Stress

If the remaining two normal and one shear stress are known in an example of plane stress, the state of stress at that point in the body on a face at any given angle to the original face can be found using Mohr's circle or its equations (figures and formulas pp. 271–273).

STRAIN, p. 274

The angle between two line segments, perpendicular before loading and meeting at the point at which strain on an element is defined, may change under load; the decrease in the angle is the shear strain (figure and formula p. 274).

Plane Strain

Equations and diagrams similar to those for plane stress (above), but substituting values for axial and shear strain, may be used to find principal strains and their orientation (formulas p. 275).

HOOKE'S LAW, p. 276

In an isotropic material, the three components of axial strain may be expressed in terms of the material's modulus of elasticity, Poisson's ratio, and the three components of normal stress; each of the three components of shear strain may be expressed in terms of the material's modulus of elasticity, Poisson's ratio, and the applicable component of shear stress (formulas p. 276).

TORSION, p. 277

This section deals with long members that twist under load; the members are commonly circular in cross-section or else hollow and thin-walled, with various cross-sections.

Circular Shafts

Using cylindrical coordinates, the product of the radius of a circular shaft and the shaft's rate of twist equals the shear strain for given point on the shaft at that angle of twist; applying Hooke's Law allows determination of shear stress, torque, and other properties (figure and formulas pp. 277–278).

Hollow, Thin-Walled Shafts

By assuming that the shear stress in the axial direction is constant throughout the wall thickness, the quantity shear flow can be defined in terms of that stress and thickness, leading to calculation of total torque (figure and formulas pp. 279–280).

BEAMS, p. 280

Shear and Moment Diagrams

Plotting shear forces and bending moments along a beam in a diagram clearly shows the maxima of these quantities; values for shear and moment may be determined by summing forces and moments about a section through the beam or by using differential relationships of shear and moment on an element of the beam, or by a combination of methods (figures and formulas pp. 281–283).

Stresses in Beams

A series of assumptions and application of Hooke's Law makes it possible to derive the normal stress at a point in a loaded beam from the product of the beam's modulus of elasticity, its curvature under load, and the distance from the point to the neutral axis; summing gives the bending moment and allows determination of the maximum bending stress (figure and formulas pp. 285–286).

Shear Stress

For a given point in a cross-section of a beam, the shear stress is the product of the shear in the beam and the moment of area above or below the given point, divided by the product of the moment of inertia of the entire beam and the thickness of the cross-section (figure and formulas pp. 286–287).

Deflection of Beams

If the slope of a deflected beam is small, the slope and deflection at a point on the beam can be found from the moment of the load about the point, the beam stiffness, and constants determined from the boundary conditions (formulas pp. 288–289).

Fourth-Order Beam Equation

The equation relating the curvature of a beam to its bending moment and stiffness, combined with the differential relationships between the shear, moment, and distributed load, leads to another means of finding slope and deflection (figure, formulas, and table pp. 290–293).

Superposition

Problems which can be understood as a combination of two problems may be solved by superposition of the solutions to those two problems.

COMBINED STRESS, p. 295

Forces may be applied to a member which result in various combinations of axial, torsional, and bending loads; each may be solved independently and the effects added.

COLUMNS, p. 297

Slender beams under high axial loads will buckle at a load related to their length and stiffness; the buckled shape depends on how they are supported (figures and formulas pp. 297–298).